

**St Stephen's School – Carramar Campus**  
**Mathematics Learning Area**

**Square pegs in round holes**

**Investigation 3AMAS**

**Take-home section—solutions**

**Part A**

**Question 4**

Mark	Description
N/A	<p>(a) The area of the triangle is <math>\sqrt{3} \times a^2 \text{ cm}^2</math></p> <p>(b) The radius of the inscribed circle is <math>\frac{a}{\sqrt{3}} \text{ cm}</math></p> <p>The radius of the circumscribed circle is <math>\frac{2a}{\sqrt{3}} \text{ cm}</math></p> <p>(c) (i) Area of the inscribed circle is <math>\frac{\pi}{3} a^2 \text{ cm}^2</math></p> <p>Percentage of material wasted is <math>\left(\sqrt{3} - \frac{\pi}{3}\right) \div \sqrt{3} \times 100 = 39.53\%</math></p> <p>(ii) Area of circumscribed circle is <math>\frac{4\pi}{3} a^2 \text{ cm}^2</math></p> <p>Percentage of material wasted is <math>\left(\frac{4\pi}{3} - \sqrt{3}\right) \div \frac{4\pi}{3} \times 100 = 58.65\%</math></p> <p>(d) (i) is more efficient.</p>

**Question 5**

Mark	Description
N/A	<p>(a) The area of the square is <math>4a^2 \text{ cm}^2</math></p> <p>(b) The radius of the inscribed circle is <math>a \text{ cm}</math></p> <p>The radius of the circumscribed circle is <math>\sqrt{2} \times a \text{ cm}</math></p> <p>(c) (i) Area of the inscribed circle is <math>\pi \times a^2 \text{ cm}^2</math></p> <p>Percentage of material wasted is <math>(4 - \pi) \div 4 \times 100 = 21.46\%</math></p> <p>(ii) Area of circumscribed circle is <math>2\pi \times a^2 \text{ cm}^2</math></p> <p>Percentage of material wasted is <math>(2\pi - 4) \div 2\pi \times 100 = 36.34\%</math></p> <p>(d) (i) is more efficient.</p>

**Question 6**

Mark	Description
N/A	<p>(a) The area of the hexagon is <math>6\sqrt{3}a^2 \text{ cm}^2</math></p> <p>(b) The radius of the inscribed circle is <math>\sqrt{3} \times a \text{ cm}</math></p> <p>The radius of the circumscribed circle is <math>2a \text{ cm}</math></p> <p>(c) (i) Area of the inscribed circle is <math>3\pi \times a^2 \text{ cm}^2</math></p> <p>Percentage of material wasted is <math>(6\sqrt{3} - 3\pi) \div 6\sqrt{3} \times 100 = 9.31\%</math></p> <p>(ii) Area of circumscribed circle is <math>4\pi \times a^2 \text{ cm}^2</math></p> <p>Percentage of material wasted is <math>(4\pi - 6\sqrt{3}) \div 4\pi \times 100 = 17.30\%</math></p> <p>(d) (i) is more efficient.</p>

**Question 7**

Mark	Description
N/A	The percentage of material wasted is greatest when the triangle is cut and least when the hexagon is cut from the circle. As the number of sides increases, the percentage of material wasted decreases.

**Part B****Question 1**

Mark	Description
N/A	<p>(a) Area of paddock = <math>24^2 = 576 \text{ m}^2</math></p> <p>Grazing area = <math>\frac{1}{4} \times 12^2 \times \pi = 113.097 \text{ m}^2</math></p> <p>Percentage area = 19.635%</p> <p>(b) Length of rope :</p> <p>Solve: <math>\frac{1}{4} \times r^2 \times \pi = \frac{576}{2}</math> for <math>r</math>.</p> <p>Length of rope = 19.149 m.</p> <p>(c) The shared area consists of 2 congruent segments. In this diagram, triangle ABC is right-angled. (The diagonals of a square are perpendicular.)</p> <p>The length of AB = <math>\frac{\sqrt{2}}{2} \times 24 \text{ m}</math></p> <p>(AB is half the diagonal of the square.)</p> <p>The length of AC = 19.149 m</p> <p><math>\angle CAB = \cos^{-1}\left(\frac{AB}{AC}\right) = \cos^{-1}\left(\frac{19.1492}{16.97056}\right)</math></p> <p>= 0.482 radians</p> <p>The area of each segment = <math>\frac{1}{2} \times 19.149^2 (0.482 - \sin 0.482) \text{ m}^2</math></p> <p>= 3.375 <math>\text{m}^2</math></p> <p>The shared area = 6.75 <math>\text{m}^2</math></p> 